Pergamon

J. Appl. Maths Mechs, Vol. 58, No. 1, pp. 11–19, 1994

0021–8928/94 $24.00+0.00

0021–8928(94)E0019-7

THE EVOLUTION OF THE ROTATIONAL MOTION OF A SATELLITE UNDER THE ACTION OF A DISSIPATIVE AERODYNAMIC MOMENT†

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(*Received* 29 *July* 1992)

The evolution of the rotation of a satellite is studied. The use of the complete formula of the dissipative aerodynamic moment reveals qualitatively new effects, distinct from those previously described [1–4]. The influence of the untwisting component of the aerodynamic moment, due to the gradient of atmospheric density along an elongated spacecraft, is also investigated.

Investigations of the effect of the dissipative aerodynamic moment (DAM) on the evolution of rotation in a satellite [1–4] have been based on an incomplete expression for the DAM, which omits the "untwisting" component. This component is due to the rotational effect of the orbital frame of reference. As a result the DAM vanishes together with the relative angular velocity of the satellite, rather than together with the absolute angular velocity, as is usually assumed in studies of the evolution of rotation [1–4].

1. A MODEL OF AERODYNAMIC ACTION

Assuming that collisions between atmospheric molecules and the surface of the satellite are absolutely inelastic, the formula for the dissipative aerodynamic moment (DAM) has the form [1, 3]

$$\mathbf{M}_2 = \oint_S \rho_a V\{[\mathbf{e}_V \times \mathbf{r}_n]((\boldsymbol{\Omega} - \boldsymbol{\omega}_c)\times \mathbf{r}_n, \mathbf{n}) + (\mathbf{e}_V, \mathbf{n})[[(\boldsymbol{\Omega} - \boldsymbol{\omega}_c)\times \mathbf{r}_n]\times \mathbf{r}_n]\}ds \tag{1.1}$$

$$\mathbf{e}_V = \mathbf{V}/V, \quad \boldsymbol{\omega} = \boldsymbol{\Omega} - \boldsymbol{\omega}_c$$

where ρ_a is the density of the atmosphere, $\mathbf{V}$ is the velocity of the satellite's centre of mass, $\boldsymbol{\Omega}$ is the absolute angular velocity, $\boldsymbol{\omega}_c$ is the angular velocity of rotation of the vector $\mathbf{V}$ in the orbital frame of reference (OFR), $\mathbf{r}_n$ denotes the radius-vector drawn from the satellite centre of mass to an element of area ds, and $\mathbf{r}$ is the normal to ds. In a circular orbit $\boldsymbol{\omega}_c$ is identical with the orbital angular velocity $\boldsymbol{\omega}_0$.

Previous research [1–4] omitted the vector $\boldsymbol{\omega}_c$. from Eq. (1.1), on the assumption that it could be ignored. This assumption is legitimate if one is studying the dynamics of the system at fairly large $\boldsymbol{\Omega}$ values. It is advisable to allow for the effect of rotation of the incident flow together with the OFR. Then $\boldsymbol{\omega}_c \neq 0$ in (1.1) and the DAM is determined by the relative angular velocity vector $\boldsymbol{\omega}$.

†*Prikl. Mat. Mekh.* Vol. 58, No. 1, pp. 13–20, 1994.

Assuming that the satellite is axisymmetric, we introduce a semi-fixed frame of reference (SFFR) with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ and origin at the satellite's centre of mass, in such a way that $\mathbf{k}$ points along the axis of symmetry, the vector $\mathbf{V}$ lies in the $(\mathbf{j}, \mathbf{k})$ plane, and moreover $(\mathbf{j}\hat{,}\mathbf{V}) \supseteq 0$. Let δ be the angle of attack: $\delta = (\mathbf{V}\hat{,}\mathbf{k})$. The absolute angular velocity of rotation of the body, $\boldsymbol{\Omega}$, and the relative angular velocity of the body in the OFR, $\boldsymbol{\omega} = \boldsymbol{\Omega} - \boldsymbol{\omega}_0$, written in the SFFR are $\boldsymbol{\Omega}(\Omega_x, \Omega_y, \Omega_z)^T$ and $\boldsymbol{\omega}(\omega_x, \omega_y, \omega_z)^T$, respectively.

Let us assume that the $\mathbf{k}$ axis is one of the principal central axes of inertia of the satellite. We also introduce a fixed frame of reference (FFR) whose unit vectors $\mathbf{i}', \mathbf{j}', \mathbf{k}$ are the principal central axes of inertia of the satellite. In this system Ω and ω are written as $\boldsymbol{\Omega}(p, q, r)^T$ and $\omega(\omega_x', \omega_y', \omega_z')^T = \boldsymbol{\Omega} - \omega_c \mathbf{e}_n$, where $\mathbf{e}_n(\beta, \beta', \beta'')^T$ are the coordinates of the unit vector normal to the orbit in the FFR. For a solid of revolution, formula (1.1) for the DAM $\mathbf{M}_2$ in the SFFR becomes [1]

$$\mathbf{M}_2 = D\boldsymbol{\omega}, \quad D = \rho_a V \begin{Vmatrix} -D_{11} & 0 & 0 \\ 0 & -D_{22} & D_{23} \\ 0 & D_{32} & -D_{33} \end{Vmatrix} \tag{1.2}$$

where $D_{ij}(\delta)\,(i,\ j = 1,\ 2,\ 3)$ are coefficients depending on the shape of the satellite and its position relative to the flow [1].

To a first approximation one can make the following assumptions about the elements of (1.2) [1]

$$D_{11} = D_{22} = \overset{0}{D}_{11}, \quad D_{23} = \overset{0}{D}_{23}\sin\delta, \quad D_{32} = \overset{0}{D}_{32}\sin\delta, \quad D_{33} = \overset{0}{D}_{33}, \quad \overset{0}{D}_{ij} = \text{const} \tag{1.3}$$

It was pointed out [4] that the approximation (1.3) does not accurately describe all the dynamical effects associated with aerodynamic dissipation. For example, according to (1.3), the angular momentum vector of a "dumbbell" (an inextensible couple of two point masses) changes its orientation owing to the DAM only along an elliptical orbit, while remaining fixed along a circular orbit. In actual fact the orientation of the angular momentum vector L evolves along both elliptical and circular orbits [4]. It should be noted, however, that, according to [4], the vector $\mathbf{L}$ should lie in the limit in the orbital plane, but that is not true. In the terminal regime, when the satellite has a low angular velocity, one can no longer omit the untwisting component of the DAM.

We shall henceforth use the exact formula for the DAM in the FFR, allowing for the effect created by rotation of the OFR

$$\mathbf{M}_2' = D'\boldsymbol{\omega}, \quad \mathbf{D}' = \rho_\pi V \Xi$$

$$\Xi = \begin{Vmatrix} -(D_{11}\cos^2\varphi + D_{22}\sin^2\varphi) & (D_{11} - D_{22})\sin\varphi\cos\varphi & D_{23}\sin\varphi \\ (D_{11} - D_{22})\sin\varphi\cos\varphi & -D(_{11}\sin^2\varphi + D_{22}\cos^2\varphi) & D_{23}\cos\varphi \\ D_{32}\sin\varphi & D_{32}\cos\varphi & -D_{33} \end{Vmatrix} \tag{1.4}$$

where φ is the angle of rotation of the FFR relative to the SFFR.

2. THE DISSIPATIVE AERODYNAMIC MOMENT IN THE FIXED FRAME OF REFERENCE

Let $(\alpha, \alpha', \alpha'')^T$ be the coordinates of the unit velocity vector $\mathbf{e}_V$ in the FFR. Then, taking into account that

$$\cos^2\varphi = \alpha'^2/\sin^2\delta, \quad \sin^2\varphi = \alpha^2/\sin^2\delta$$

one can use (1.4) to express the DAM in terms of the FFR for specific bodies.

For example, for a sphere of radius R

$$\mathbf{M}_2' = D_s'\boldsymbol{\omega}, \quad D_s' = \rho_a V C_1 \begin{Vmatrix} -(3-\alpha^2) & \alpha\alpha' & \alpha\alpha'' \\ \alpha\alpha' & -(3-\alpha'^2) & \alpha'\alpha'' \\ \alpha\alpha'' & \alpha'\alpha'' & -(3-\alpha''^2) \end{Vmatrix}, \quad C_1 = \frac{\pi}{4} R_s^4 \tag{2.1}$$

For a "dumbbell," i.e. a couple of two-point masses m_1 and m_2 of length l, with its axis in the $\mathbf{k}$ direction, we have

$$\mathbf{M}_2' = D_1'\boldsymbol{\omega}, \quad D_1' = \rho_a V C_\gamma \begin{Vmatrix} -(1+\alpha'^2) & \alpha\alpha' & 0 \\ \alpha\alpha' & -(1+\alpha^2) & 0 \\ 0 & 0 & 0 \end{Vmatrix},$$

$$C_\gamma = \frac{\pi}{2}\pi l^2 \frac{m_1 m_2}{(m_1+m_2)^2}(R_1^2 + R_2^2) \tag{2.2}$$

where R_1 and R_2 may be treated as the radii of "virtual" spheres at the ends of the dumbbell. In a symmetric dumbbell $m_1 = m_2 = m$, $R_1 = R_2 = R$.

For a symmetric three-dimensional dumbbell (a body made up of three identical symmetric dumbbells with intersecting axes, in such a way that the point masses lie at the vertices of an octahedron), we have an expression that differs from (2.1) only in that C_1 is replaced by C_γ. We may therefore expect that the DAM of a three-dimensional dumbbell with non-identical elements will be a good approximation for the DAM of a body with a surface of fairly general shape. Thus, for a "doubly symmetric" dumbbell (two of the three dumbbells are symmetric and identical and the third is symmetric but different from the first two) we have the following superposition formula (corresponding dimensions $lb^{1/2}$ and l)

$$D_{32}' = \rho_a V C_\gamma \begin{Vmatrix} -b(1+\alpha'^2)-(1+\alpha''^2) & b\alpha\alpha' & \alpha\alpha'' \\ b\alpha\alpha' & -b(1+\alpha^2)-(1+\alpha''^2) & \alpha'\alpha'' \\ \alpha\alpha'' & \alpha'\alpha'' & -(2+\alpha'^2+\alpha''^2) \end{Vmatrix}, \tag{2.3}$$

$$C_\gamma = \pi/2R_2^2 l_2^2, \quad b = R_1^2 l_1^2/(R_2^2 l_2^2)$$

where R_2 and l_2 are the elements of the two "transverse" dumbbells, and R_1 and l_1 are those of the "axial" dumbbell. The tensor (2.3) is readily expressed as the superposition of the tensor for a sphere with $C_\gamma = \pi/2R_2^2 l_2^2$ and the tensor for an axial dumbbell with $C_\gamma = \pi/2R_2^2 l_2^2 (R_2^2 l_1^2)/(R_2^2 l_2^2) - 1$.

We may expect this superposition to be a satisfactory approximation for the DAM of any doubly symmetric body, whether elongated $(R_1 l_1 > R_2 l_2)$ or oblate $(R_1 l_1 < R_2 l_2)$.

Throughout, $\boldsymbol{\omega} = \boldsymbol{\Omega} - \omega_c(\beta, \beta', \beta'')^T$.

3. THE EVOLUTIONARY VARIABLES

We shall describe the perturbed motion of the body relative to its centre of mass in evolutionary variables [1]: (L, ρ, σ), $(\varphi, \psi, \vartheta)$, where L is the magnitude of the angular momentum vector $\mathbf{L}$, ρ is the angle between $\mathbf{L}$ and the normal $\mathbf{n}$ to the orbital plane, σ is the angle between the projection of $\mathbf{L}$ onto the orbital plane and the perigean radius-vector of the orbit $\mathbf{r}_\pi$ (Fig. 1a), $(\varphi_1, \psi, \vartheta)$ are the Euler variables, which describe the orientation of the satellite relative to L, and υ is the angle between $\mathbf{L}$ and the principal central axis of inertia $\mathbf{k}$ (Fig. 1b). Since our satellite is dynamically symmetric (k is the dynamical axis of symmetry), the angle φ_1 becomes a cyclic variable. The angle of precession ψ is a "fast" variable, and the variables $L, \rho, \sigma, \vartheta$ are "slow".

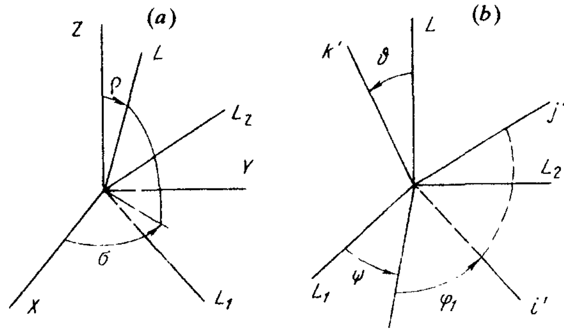

Fig. 1.

To develop the evolution equations, we have to express the perturbed DAM, originally in the FFR, in terms of the fixed "perigean" frame of reference $OXYZ$ (the $\mathbf{Y}$ axis points along the normal to the orbit and the $\mathbf{Z}$ axis along $\mathbf{r}_\pi$), and then average over the fast variable ψ and the true anomaly ν – the angle between $\mathbf{r}_\pi$ and the radius-vector of the satellite.

4. THE EVOLUTION EQUATIONS FOR A SPHERE

Suppose e is the eccentricity of the orbit, a is the semi-major axis of the orbit, P is its focal parameter, $P = a(1-e^2)$, and μ is the gravitational constant.

Applying the procedure outlined above to (2.1), we obtain the following evolution equations

$$\dot{\Omega} = -A\Omega\left(\frac{\sin^2\vartheta}{A} + \frac{\cos^2\vartheta}{C}\right)[3N_0 - \sin^2\rho(J_2\sin^2\sigma + J_3\cos^2\sigma)] + 3\upsilon_1\cos\rho$$

$$\Omega\dot{\rho} = \frac{1}{2}A\Omega\left(\frac{\sin^2\vartheta}{A} + \frac{\cos^2\vartheta}{C}\right)(J_2\sin^2\sigma + J_3\cos^2\sigma)\sin 2\rho - 3\upsilon_1\sin\rho \tag{4.1}$$

$$\Omega\dot{\sigma} = \frac{1}{2}A\Omega\left(\frac{\sin^2\upsilon}{A} + \frac{\cos^2\vartheta}{C}\right)(J_2 - J_3)\sin 2\sigma$$

$$\dot{\vartheta} = \frac{1}{4}A\left(\frac{1}{C} - \frac{1}{A}\right)[5N_0 + \sin^2\rho(J_2\sin^2\sigma + J_3\cos^2\sigma]]\sin 2\vartheta$$

where

$$\Omega = L/(A\omega_0), \quad \omega_0 = \mu^{1/2}a^{-3/2}, \quad N_0 = J_2 + J_3$$

$$J_2 = \frac{1}{2\pi}\int_0^{2\pi}\bar{\rho}(e+\cos\nu)^2 f_{-2,-1/2}(\nu)d\nu$$

$$J_3 = \frac{1}{2\pi}\int_0^{2\pi}\bar{\rho}\sin^2\nu f_{-2,-1/2}(\nu)d\nu$$

$$\upsilon_3 = \frac{1}{2\pi}(1-e^2)^{-3/2}\int_0^{2\pi}\bar{\rho}f_{1,-1/2}(\nu)d\nu$$

$$f_{n,m} = (1+e\cos\nu)^n(1+2e\cos\nu+e^2)^m$$

The dot denotes differentiation with respect to the new dimensionless time τ, $\bar{\rho} = \rho_a/\rho_\pi$ is a dimensionless density function [1], p_π is the density at the perigee of the orbit, $d\tau = A^{-1}p_\pi C_1 dv$, and A and C are the components of the inertia tensor of the satellite: $J = \mathrm{diag}\,(A, A, C)$. The first three equations without the terms in υ_1 are identical, apart from the notation, with the equations of [4]. Numerically computed curves of the functionals N_1 and υ_1, plotted against the eccentricity for a standard atmosphere, are shown in Fig. 2.

To fix our ideas, let us assume that $A > C$. It then follows from the last equation of (4.1) that $\vartheta \to \pi/2$, $\tau \to \infty$. The rotational motion of the sphere tends to a purely axial motion about the transverse axis. Put $\vartheta = \pi/2$. Then Eqs (4.1) reduce to a linear system with separated variables, via the following substitutions:

$$\Omega_n = \Omega\cos\rho, \quad \Omega_\pi = -\Omega\sin\rho\cos\sigma, \quad \Omega_\tau = -\Omega\sin\rho\sin\sigma$$

We obtain

$$\begin{aligned}&\dot{\Omega}_n = -\alpha\Omega_n + 3\upsilon_1, \quad \dot{\Omega}_\pi = -b\Omega_\pi, \quad \dot{\Omega}_\tau = -c\Omega_\tau,\\ &\alpha = 3N_0, \quad b = 3N_0 - J_3, \quad c = -(3N_0 - J_0)\end{aligned} \tag{4.2}$$

The system has the following first integrals

$$C_{e1} = \Omega_\pi^{a/b}/(\Omega_n - \nu_1/N_0), \quad C_{e2} = \Omega_\pi^c/\Omega_\tau^b \tag{4.3}$$

It follows from the second of these integrals that in a circular orbit $(J_2 = J_3 = 1/2)\mathrm{ctg}\,\sigma = C_{e2}^{2/5}$, i.e. in a circular orbit $\mathbf{\Omega}$ evolves in a constant plane $(\mathbf{\Omega}, \mathbf{n})$ (the axes may be so chosen that $\sigma = 0$).

Integrating system (4.2), we see that along an elliptical orbit $\mathrm{tg}\,\sigma = \mathrm{tg}\,\sigma_0 \exp(J_2 - J_3)\tau$. Since $J_2 < J_3^1$,† it follows that $\sigma \to 0$ as $\tau \to \infty$. Set $\sigma = 0$. It is clear from the above that the partial motion $\vartheta = \pi/2$, $\sigma = 0$ is stable and described by the two variables Ω and ρ. The solid curves in Fig. 3 are a phase portrait of the evolution of the motion of a spherical satellite in an elliptical orbit in the parameter plane (Ω, ρ). Values of Ω are plotted along the ordinate axis in the range $0 < \Omega < 1$, and values of Ω^{-1} for $\Omega > 1$. This inversion of the parameter plane [5] enables the phase portrait to be drawn in its entirety.

The dashed curves in Fig. 3 represent the special case of a circular orbit. This phase portrait is structurally analogous to that of the tidal evolution of a celestial body along a circular orbit. While the value of the limit point of the evolution is the same $(\Omega^* = 1, \ \rho = 0)$, a change occurs in the curve of the extrema with respect to the inclination ρ: instead of the set $\{\Omega\cos\rho = 2\}$, we have the set $\gamma_m\{\Omega\cos\rho = 6\}$, represented in Fig. 3 by the dash–dot curve. It is clear that the motion tends in the limit to rotation about an axis orthogonal to the orbital plane at an angular velocity equal to the orbital velocity. This obviously implies that the satellite takes up a relative equilibrium position in the OFR, which seems physically natural. In a previous treatment, however, the limiting values were $\Omega = 0$, $\rho = \pi/2$ [4]. The discrepancy is due to the incomplete use of the model of the DAM in [1–4], as already pointed out in [1].

The fundamental difference between evolution along an elliptical orbit (Fig. 3) and evolution along a circular orbit is the increase in the magnitude of the satellite's angular velocity of rotation at the limit point of the evolution: $\{\Omega^* = \upsilon_1/N_0, \ \rho = 0\}$. The quantity $\Omega^*(e)$ increases together with e, as shown in Fig. 2.

† BELETSKII V. V. and GRUSHEVSKII A. V., The evolution of the axial rotations of an artificial satellite under the action of a dissipative aerodynamic moment. Preprint No. 75, Inst. Prikl. Mat. im. Keldysh, Akad. Nauk SSSR, Moscow, 1991.

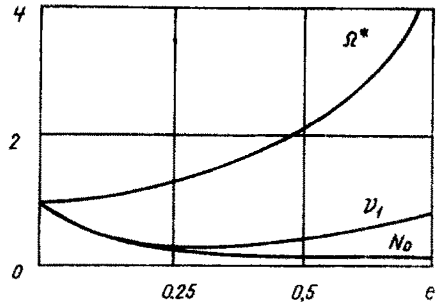

Fig. 2.

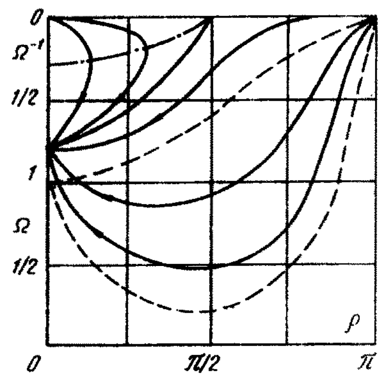

Fig. 3.

5. THE EVOLUTION OF THE MOTION OF A DUMBBELL-SHAPED SATELLITE

For a dumbbell-shaped satellite in an elliptical orbit, the use of formula (2.2) and the procedure outlined above, including averaging of the evolution equations, yields

$$\dot{\Omega} = -\Omega\frac{\sin^2\vartheta}{2}[3N_0 - \sin^2\rho(J_2\sin^2\sigma + J_3\cos^2\sigma)] + \frac{3}{2}\upsilon_1\cos\rho$$

$$\Omega\dot{\rho} = \Omega\left(\frac{\sin^2\vartheta}{4}(J_2\sin^2\sigma + J_3\cos^2\sigma)\sin 2\rho -\right.$$

$$\left. -\sin\rho\left[\upsilon_1 + \left(1 - \frac{3}{2}\sin^2\vartheta\right)(j_2\cos^2\sigma + j_3\sin^2\sigma)\right]\right) \tag{5.1}$$

$$\Omega\dot{\sigma} = \Omega\frac{1}{2}\sin 2\sigma\left(\frac{1}{2}(J_2 - J_3)\sin^2\vartheta + (j_2 - j_3)\cos\rho\left(1 - \frac{3}{2}\sin^2\vartheta\right)\right)$$

$$\dot{\vartheta} = -\frac{1}{4}\sin 2\vartheta([3N_0 - \sin^2\rho(J_2\sin^2\sigma + J_3\cos^2\sigma)] - 3\upsilon_1\cos\rho)$$

The variable Ω and new dimensionless time τ are defined just as in (4.1). The coefficients J_1, J_2, $N_0 = J_1 + J_2$, υ_1 have the same form (4.1) as for a sphere, and the new coefficients are given by the following formulae

$$j_2 = \frac{1}{2\pi}(1-e^2)^{-3/2}\int_0^{2\pi}\bar{\rho}(e + \cos\nu)^2 f_{1,-3/2}(\nu)d\nu$$

$$j_3 = \frac{1}{2\pi}(1-e^2)^{-3/2}\int_0^{2\pi}\bar{\rho}\sin^2\nu f_{1,-3/2}(\nu)d\nu$$

Obviously, $\upsilon_1 = j_2 + j_3$.

For a dumbbell, the equation for ϑ is purely formal, since the integral $L\cos\upsilon = C_L = 0$ vanishes and $\vartheta = \pi/2$. But the formal equation for ϑ is necessary, for example, to construct superposition equations for a three-dimensional dumbbell. For the readily investigated case of a one-dimensional dumbbell one must put $\vartheta = \pi/2$ in (5.1), which gives

$$\begin{aligned}
\dot{\Omega} &= -\frac{\Omega}{2}[3N_0 - \sin^2\rho(J_2\sin^2\sigma + J_3\cos^2\sigma)] + \frac{3}{2}\upsilon_1\cos\rho \\
\Omega\dot{\rho} &= \frac{\Omega}{4}\left(J_2\sin^2\sigma + J_3\cos^2\sigma)\sin 2\rho - \frac{1}{A}\sin\rho\left[\upsilon_1 - \frac{1}{2}(j_2\cos^2\sigma + j_3\sin^2\sigma)\right]\right) \\
\Omega\dot{\sigma} &= \frac{\Omega}{4}\sin 2\sigma((J_2 - J_3) - (j_2 - j_3)\cos\rho)
\end{aligned} \tag{5.2}$$

For a circular orbit $(J_2 = J_3 = j_2 = j_3 = 1/2)$, obviously

$$\sigma = \text{const}$$

$$\dot{\Omega} = -\frac{1}{2}\left(\left(3 - \frac{1}{2}\sin^2\rho\right)\Omega - 3\cos\rho\right), \quad \Omega\dot{\rho} = \frac{1}{4}\sin\rho(\Omega\cos\rho - 3) \tag{5.3}$$

The evolution equations of the dumbbell have the same structure as the corresponding equations for a sphere (5.1). The phase portrait for rotational motion of a dumbbell in the (Ω, ρ) plane (Fig. 4) also retains its similarity to the spherical case (Fig. 3). The curve of maxima with respect to ρ for a circular orbit has the form $\gamma_{m2}\{\Omega\cos\rho = 3\}$, represented by the dash–dot curve in Fig. 4. As in the spherical case, the limiting singular point of the evolution is determined by the values $\rho^* = 0$, $\Omega^* = 1$ for a circular orbit and $\rho^* = 0$, $\Omega^* = \nu_1/N_0$ for an elliptical orbit.

It is noteworthy that Eqs (5.2) do not lead to a linear system in the same way that Eqs (4.1) led to system (4.2). Accordingly, no first integrals of type (4.3) have been observed for Eqs (5.3).

The most interesting effects observed in the evolution of rotational motions of a spherical satellite and a dumbbell-shaped satellite due to the "untwisting" component of the DAM may be listed as follows:

1. All motions (including initially retrograde motions) tend to forward rotation at zero inclination. In a circular orbit the limiting angular velocity of all motions equals the orbital velocity.
2. Trajectories exist with initially small inclination which increases during the intermediate stage of the evolution, reaching a value of $\pi/2$.
3. The angular velocity of the satellite may decrease to less than the orbital velocity, but it will subsequently return to the orbital value.

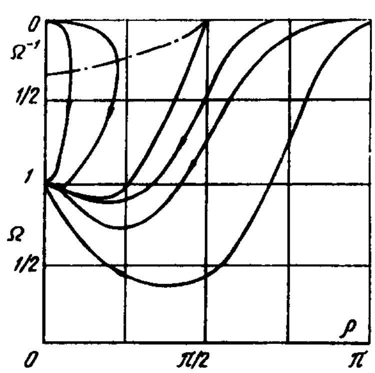

Fig. 4.

6. THE AEROGRADIENT EFFECT

The complete aerodynamic moment for a dumbbell may be expressed as a sum

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

where $\mathbf{M}_2$ is the DAM and $\mathbf{M}_1$ is the moment of the pressure forces of the aerodynamic flow:

$$\mathbf{M} = cv_0^2 l\mathbf{k} \times \mathbf{e}_V, \quad c = \pi(\rho_1 R_1^2/m_1 - \rho_2 R_2^2/m_2)m_1 m_2/(m_1 + m_2)^2 \tag{6.1}$$

Here $\mathbf{e}_V$ is the unit vector in the direction of the forward velocity of the centre of mass of the dumbbell, $\mathbf{k}$ is the unit vector along the dumbbell axis (directed from m_1 to m_2), m_i and R_i are the masses and radii of the virtual spheres at the ends of the dumbbell, and ρ_1 and ρ_2, denote the (generally different) atmospheric densities at the current positions of the masses m_1 and m_2.

In computing the DAM $\mathbf{M}_2$ one assumes $\rho_1 = \rho_2$, an assumption fully justified by the neglect of the relatively small interactive effect of the dissipation and the atmospheric density gradient. That is not the case with regard to the moment $\mathbf{M}_1$ (6.1). For the very elongated objects that have been studied in the literature, the aerogradient effect may be considerable [3, 5].

Let $\mathbf{e}_0$ be the unit vector in the direction of the radius-vector of the orbit of the dumbbell's centre of mass, and let $\mathbf{r}_1 = -m_1(m_1 + m_2)^{-1} l\mathbf{k}$, $r_2 = -m_2(m_1 + m_2)^{-1} l\mathbf{k}$ be the vector from the centre of mass to the end of the dumbbell. We put

$$\rho_1 = \rho_0 \exp(-r_1 \bar{e}_0 / H), \quad \rho_2 = \rho_0 \exp(-r_2 \bar{e}_0 / H) \tag{6.2}$$

where ρ_0 is the density of the atmosphere at the level of the centre of mass, and H is the so-called "altitude of the uniform atmosphere". Assuming that $l/H \ll 1$ and expanding the exponential functions in (6.2) in powers of this parameter, we obtain from (6.1), retaining the first two terms

$$\begin{gathered}\mathbf{M}_1 = \mathbf{M}_0 + \mathbf{M}_H \\ \mathbf{M}_0 = c_0 \rho_0 v_0^2 \mathbf{k} \times \mathbf{e}_v \quad \mathbf{M}_H = c_H \rho_0 v_0^2 (\mathbf{k}\mathbf{e}_0)\mathbf{k} \times \mathbf{e}_v \\ c_0 = \pi l(m_2 R_1^2 - m_1 R_2^2)/(m_1 + m_2), \quad C_H = \pi l(l/H)(m_2^2 R_1^2 + m_1^2 R_2^2)/(m_1 + m_2)^2\end{gathered} \tag{6.3}$$

The effect of a conservative aerodynamic moment $\mathbf{M}_0$ of the type (6.3) on the evolution of the rotation and orientation of a satellite has been studied in detail [1, 3]. The rate of evolution (on the average) is proportional to $\cos\vartheta$, and therefore the motion of even an asymmetric dumbbell is not affected by $\mathbf{M}_0$ (because in a dumbbell, by definition, $\cos\vartheta = 0$). For a symmetric dumbbell, naturally $\mathbf{M}_0 = 0$.

The aerogradient moment $\mathbf{M}_H$ has an untwisting effect and, as has been shown, produces on the average an evolution that is qualitatively similar to that produced by the untwisting component of the DAM (2.2). Thus, for a circular orbit, instead of the evolution equations (5.3), we get

$$\begin{gathered}\dot{\Omega} = -\frac{1}{2}\left(\left(3 - \frac{1}{2}\sin^2\rho\right)\Omega - \left(3 + \kappa\frac{P}{H}\cos\rho\right)\right) \\ \Omega\dot{\rho} = \frac{1}{4}\sin\rho\left(\Omega\cos\rho - \left(3 + \kappa\frac{P}{H}\right)\right) \\ \Omega = \frac{L}{A\omega_0}, \quad \kappa = \frac{m_2^2 R_1^2 + m_1 R_2^2}{m_1 m_2 (R_1^2 + R_2^2)}, \quad A = l^2\frac{m_1 m_2}{m_1 + m_2}, \quad d\tau = \frac{\rho P\pi(R_1^2 + R_2^2)\omega_0}{m_1 + m_2}dt\end{gathered} \tag{6.4}$$

In particular, in a symmetric dumbbell $\kappa = 1$.

Equations (5.3) are structurally identical to (5.3) and yield the same qualitative pattern of the motion, including the phase portrait in the (Ω, ρ) plane, which is similar to that shown in Fig. 4. Only the limit point $\rho=0$; $\Omega^*=1+\kappa P/(3H)$ corresponds to a considerably higher angular velocity (recall that P denotes the radius of the circular orbit of the centre of mass of the dumbbell). For a symmetric dumbbell, the limiting angular velocity $\Omega^*=1+\kappa P/(3H)$ does not depend at all on the parameters of the dumbbell, but only on the ratio of the orbit radius to the adjusted altitude of a uniform atmosphere (estimates of the limiting angular velocity in dimensional form give 1.5 to 3 deg/s). The rate of evolution, as is clear from (6.4), is independent of the length l of the dumbbell; this is natural, since the moments of the active forces and the moment of inertia of the dumbbell are both of the order of l^2.

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Translated by D.L.